

Ex: $y'' - 2y' + y = te^t$

Initial guess: $y = (At + B)e^t = At^1e^t + Be^t$

↑ ↑
appear in the complementary sol.

Second guess: $y = (At^2 + Bt + C)e^t = At^2e^t + \underbrace{Bte^t + Ce^t}_{\text{appear in comp. sol.}}$

Refine: $y = At^2e^t$

$$y'' - 2y' + y = \dots = 2Ae^t \neq te^t$$

Third guess:

$$y = (At^3 + Bt^2 + Ct + D)e^t \\ = At^3e^t + Bt^2e^t + \underbrace{Cte^t + De^t}_{\text{appear in comp. sol.}}$$

Refine: $y = At^3e^t + Bt^2e^t$

$$y'' - 2y' + y = 6Ate^t + 2Be^t$$

$$\begin{cases} A = 1/6 \\ B = 0 \end{cases}$$

$$y_p = \frac{1}{6} t^3 e^t$$

Textbook explanation:

$$y = (At + B)e^t$$

$$\rightarrow y = t(At + B)e^t = (At^2 + Bt)e^t$$

$$\rightarrow y = t^2(At + B)e^t = (At^3 + Bt^2)e^t \\ = At^3e^t + Bt^2e^t$$